LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIFTH SEMESTER – **NOVEMBER 2024**

UST 5501 – APPLIED STOCHASTIC PROCESSES

Date: 07-11-2024 Dept. No. Max. : 100 Marks

Time: 09:00 am-12:00 pm

	SECTION A - K1 (CO1)
	Answer ALL the Questions **(5 x 1 = 5)**
1.	**Define the following**
a)	Stochastic process.
b)	Transition Probability matrix.
c)	Poisson process.
d)	Renewal period.
e)	Branching process.
2.	**MCQ- Choose the correct option** **(5 x 1 = 5)**
a)	Le$\{X(t), t \geq 0\}$ be stochastic process with independent increments. Then, which of the following is always TRUE? (a) $\{X(t), t \geq 0\}$ is a Markov Process (b) $\{X(t), t \geq 0\}$ need not be a Markov Process (c) $\{X(t), t \geq 0\}$ is a wide sense stationary stochastic process (d) $\{X(t), t \geq 0\}$ is a strict-sense stationary stochastic process
b)	Every state can be reached from every other state, the Markov chain is said to be (a) Homogeneous (b) Reducible (c) Irreducible (d) Recurrent
c)	Let $\{X(t), t \geq 0\}$ is a Poisson process, then the current life follows (a) Exponential distribution (b) Truncated exponential distribution (c) Poisson distribution (d) Normal distribution.
d)	Let $\{N(t), t \geq 0\}$ be a renewal process with inter arrival distribution U(0,1). i.e., $X_i \ U(0,1)$. Let m(t) denote the renewal function of $\{X(t), t \geq 0\}$. Then, which of the following is True for $t \in [0,1]$? $a.\, m(t) = e^t b.\, m(t) = e^{-t}$ $c.\, m(t) = e^t - 1 d.\, m(t) = e^t + 1$
e)	Which one of the following is not true in branching process (a) $Var(X_n) = n\sigma^2$ (b) $E(X_1) = m$ (c) $E(X_n) = m^n$ (d) $Var(X_{n+1}) = n\sigma^2$
	SECTION A - K2 (CO1)
	Answer ALL the Questions
3.	**True or False** **(5 x 1 = 5)**
a)	Random processes or Random variable are same (True /false).

b)	Markov Chains are stochastic processes with no memory (True /false).
c)	Poisson process is the basic example of a renewal process (True /false)
d)	A Markov renewal process is a generalization of a renewal process that the sequence of holding times is not independent and identically distributed (True /False).
e)	In Branching process we can assume that the process starts with a single ancestor (True/False).
4.	**Fill in the blanks** **(5 x 1 = 5)**
a)	A _________________is the time evolution of a random variable or a collection of random variables.
b)	A discrete time state space stochastic process $\{X_n, n \geq 0\}$ is called ______________________.
c)	The ______________is a counting that counts the number of occurrences of some specific event through time.
d)	The sequence of the time of occurrences of the event S_n, is called the ________________.
e)	In Branching process Var(X1) is ---------------------------.
SECTION B - K3 (CO2)	
Answer any TWO of the following **(2 x 10 = 20)**	
5.	Explain different classification of stochastic process by giving examples.
6.	State and prove Chapman-Kolmogorov equation.
7.	Explain Type I counter model with the necessary diagram.
8.	Explain age and block replacement policies.
SECTION C – K4 (CO3)	
Answer any TWO of the following **(2 x 10 = 20)**	
9.	Show that one-dimensional random walk is recurrent.
10.	State and prove Yule-Furry process.
11.	State and prove the elementary renewal theorem.
12.	In branching process prove that $\varphi_{n+1}(s)=\varphi_n(\varphi(s))=\varphi(\varphi_n(s))$.
SECTION D – K5 (CO4)	
Answer any ONE of the following **(1 x 20 = 20)**	
13.	a) Derive the differential equations for a pure birth process. (10) b) Given TPM of a Markov chain $$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \left\Vert\begin{array}{ccccc} 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{5} & 0 & \frac{3}{5} \end{array}\right\Vert \end{array}$$ Draw transition diagram of TPM. Check whether state two and state three is recurrent or transient. (10)
14.	Prove that Poisson process can be viewed as a renewal process.
SECTION E – K6 (CO5)	
Answer any ONE of the following **(1 x 20 = 20)**	
15.	Derive forward and backward Kolmogorov differential equations for birth and death process.
16.	a) If m=E(X1) = ΣkPk and σ2 = Var(X1) then prove that E{Xn} = mn and V(Xn)={mn-1(mn -1)/(m-1)}σ2, if m ≠1 and V(Xn)=nσ2 , if m=1.

	(12)
	b) Define ultimate extinction. Prove that if $m \leq 1$, the probability of ultimate extinction is 1. (8)

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